Errors during the measurement process can be divided into two groups

- Systematic Errors
- Random Errors

Systematic Errors

Errors in the output readings of a measurement system that are consistently on one side of the correct reading

Two major sources

- System disturbance during measurement
- Effect of environmental changes

other sources

- Use of uncalibrated insturements
- drift in instrument characteristics

Random Errors are perturbations of measurement either side of the true value caused by random and unpredictable effects

- Arise by human factor
- Electrical noise is also source of Random errors

Random errors can be overcome by taking the same measurement a number of times and extracting a value some statistical techniques

STATISTICAL ANALYSIS OF MEASUREMENTS

Random Errors in measurements are caused by variations in the measurement system. Errors can be eliminated by calculating the average of a number of repeated measurements.

Mean and median values

For any set of n measurements $x_1, x_2, \ldots x_n$ of a constant quantity, the most likely true value is the mean

$$x_{mean} = \frac{x_1 + x_2 + \cdots x_n}{n}$$

The median is the middle value when the measurements in the data set are written down in ascending order of magnitude.

For a set of n measurements $x_1, x_2 \cdots x_n$ of a constant quantity, written down in ascending order of magnitude, the mean value is given by

$$X_{median} = \frac{x_{n+1}}{2}$$

For set of 9 measurements $x_1\ x_2 \cdots x_9$

median value is X_5

For set of 10 measurements $x_1, x_2 \cdots x_{10}$

median value is $\frac{(x_5 + x_6)}{2}$

x_5 and x_6 → two centre values

Ex: Length of a iron bar is measured by a number of different observers and the following set of 11 measurements are recorded (units mm)

Measurement set A

398 420 394 416 404 408 400 420 396 413 430

$$X_{mean} = \frac{398 + 420 + \cdots + 413 + 430}{11} = 409 \text{ mm}$$

394	396	398	400	404	408	413	416	420	420	430
X_1	X_2	X_3	X_4	X_5	X_6	X_7	X_8	X_9	X_{10}	X_{11}

Median = 408 mm

→ Ascending order of magnitude

$$X_{\frac{n+1}{2}} = X_{median} = X_{\frac{12}{2}} = X_6$$

Measurement set B

(4)

409 406 402 407 405 404 407 404 407 407 408

$$x_{mean} = \frac{409 + 406 + \dots + 407 + 408}{11} = 406 \text{ mm}$$

402 404 404 405 406 $\boxed{407 \downarrow x_6}$ 407 407 407 408 409

$x_1 \rightarrow$ x_{11}

$$x_{median} = x_{\frac{n+1}{2}} = x_{\frac{12}{2}} = x_6$$

$$x_{median} = 407$$

which measurement set (Set A or Set B) is most confident?

In set B measurements are much close together. (5)

Spread between the smallest and largest value

Spread B = 409 − 402 = 7mm

In set A Spread A = 430 − 394 = 36 mm

Smaller the spread of measurements, the more confidence in the mean or median value calculated

Spread B < Spread A

So measurement set B is more reilable

Let increase the number of measurements by (6)
extending set B to 23 measurements

Measurement set C

409 406 402 407 405 404 407 404 407 407 408 406 410
406 405 408 406 409 406 405 409 406 407

$$X_{mean} = \frac{X_1 + X_2 + \dots X_{23}}{23} = 406.5mm$$

$$X_{median} = \frac{X_{23+1}}{2} = X_{12} = 406 mm$$

As number of measurements increases the difference between the mean and the median value becomes very small

For set B { Mean = 406 mm
Median = 407 mm } less close

Fort Set C { mean = 406,5 mm
median = 406 mm } more close

Standart deviation and Variance

Starting point is to calculate the deviation (error) d_i of each measurement X_i from the mean value

$$d_i = X_i - X_{mean}$$

The Variance (V) is given

$$V = \frac{d_1^2 + d_2^2 \dots + d_n^2}{n-1}$$

The standart deviation (σ) is the square root of the variance (8)

$$\sigma = \sqrt{V} = \sqrt{\frac{d_1^2 + d_2^2 + \cdots + d_n^2}{n-1}}$$

<u>For measurement set A</u>

Measurement	398	420	394	416	404	408	400	420	396	413	430
Deviation from mean	−11	+11	−15	+7	−5	−1	−9	+11	−13	+4	+21
$(\text{deviations})^2$	121	121	225	49	25	1	81	121	169	16	441

$\sum (\text{deviations})^2 = 1370$ $\quad n$ = number of measurements = 11

$$V = \frac{\sum (\text{deviations})^2}{n-1} = \frac{1370}{10} = 137$$

$X_{\text{mean}} = 409$ mm

$$\sigma = \sqrt{V} = 11.7$$

For measurement set B

Measurement	409	406	402	407	405	404	407	404	407	407	408
Deviation from mean	+3	0	−4	+1	−1	−2	+1	−2	+1	+1	+2
$(\text{deviations})^2$	9	0	16	1	1	4	1	4	1	1	4

$$\sum(\text{deviations})^2 = 42 \qquad n = 11$$

$$V = \frac{\sum(\text{deviations})^2}{11-1} = 4{,}2 \qquad \sigma = \sqrt{V}$$

$$\sigma = \sqrt{4{,}2} = 2{,}05 \qquad X_{mean} = 406\,mm.$$

For measurement Set C

$V = 3{,}53$ $\sigma = 1.88$ $X_{mean} = 406{,}5mm$

* as V and σ decrease for a measurement set, we express greater confidence that the calculated mean or median value is close to the true value. i.e. Averaging process has reduced the random error value close to zero

* V and σ get smaller as the number of measurements increases, confirming that the confidence in the mean value increases as the number of measurements increases,